Consider the polar equation $r = 2 + 2\sin 2\theta$.

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POLAR AXIS POLE POLAR AXIS

The following symmetry tests all fail: $(r, -\theta)$, $(-r, \theta)$ and $(-r, \pi - \theta)$

[a] Is the graph symmetric with respect to the polar axis ? **State your conclusion clearly.**

(1) NO CONCLUSION, (BOTH TESTS FAIL)

[b] Is the graph symmetric with respect to $\theta = \frac{\pi}{2}$? **State your conclusion clearly.** NO CONCLUSION (1)

$(r, -\theta)$ $-r = 2 + 2\sin 2(-\theta)$ (½)

$-r = 2 - 2\sin 2\theta$

$r = -2 + 2\sin 2\theta$ (½)

$(r, \pi - \theta)$ $r = 2 + 2\sin 2(\pi - \theta)$ (½)

$r = 2 + 2\sin(2\pi - 2\theta)$

$r = 2 + 2[\sin 2\pi \cos 2\theta - \cos 2\pi \sin 2\theta]$

[c] Is the graph symmetric with respect to the pole ? **State your conclusion clearly.** $r = 2 - 2\sin 2\theta$, (1)

$(r, \pi + \theta)$ $r = 2 + 2\sin 2(\pi + \theta)$ (½)

$r = 2 + 2\sin(2\pi + 2\theta)$

$r = 2 + 2[\sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta]$

(1) $r = 2 + 2\sin 2\theta$, SYMMETRIC (1)

[d] Based on the symmetry tests, what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in [0, \pi]$ OR $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ (1) FOR EITHER INTERVAL

[e] Find the zeros of the graph in the minimum interval from [d]
(ie. for what values of θ in the minimum interval does the graph pass through the pole ?).

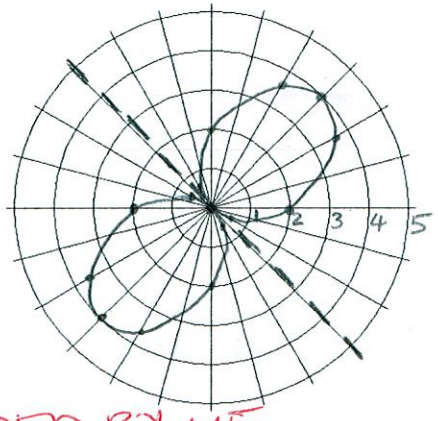
$0 = 2 + 2\sin 2\theta$

(½) $\sin 2\theta = -1$ (½) FOR EITHER 2θ

$2\theta = \frac{3\pi}{2}$ OR $-\frac{\pi}{2}$

$\theta = \frac{3\pi}{4}$ OR $-\frac{\pi}{4}$ (1) FOR EITHER θ

[f] Find the value of r for all the common values of θ in the minimum interval. Plot those points. Connect the points into a curve. Reflect that part of the curve using the results of the symmetry tests in [a], [b] and [c] to draw the complete graph.



CALCULATE THE *r* –VALUES ON SCRATCH PAPER ON THE BACK. WRITE THE POLAR COORDINATES HERE.

$(2-\sqrt{3}, -\frac{\pi}{6})$ $(2, 0)$ $(2-\sqrt{3}, \frac{2\pi}{3})$

$(0, -\frac{\pi}{4})$ $(2+\sqrt{3}, \frac{\pi}{6})$ $(0, \frac{3\pi}{4})$

$(2-\sqrt{3}, -\frac{\pi}{3})$ $(4, \frac{\pi}{4})$ $(2-\sqrt{3}, \frac{5\pi}{6})$

$(2, -\frac{\pi}{2})$ $(2+\sqrt{3}, \frac{\pi}{3})$ $(2, \pi)$

$(2, \frac{\pi}{2})$

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Consider the conic with polar equation $r = \dfrac{28}{5 - 9\sin\theta}$.

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[a] What is the type of the conic ? **Justify your answer clearly.**

$r = \dfrac{\frac{28}{5}}{1 - \frac{9}{5}\sin\theta}$ $\quad e = \frac{9}{5} > 1$ (1) $\quad$ HYPERBOLA (1)

[b] What is the equation of the directrix ?

$ep = \frac{28}{5}$

$\frac{9}{5}p = \frac{28}{5} \longrightarrow p = \frac{28}{9}$

$y = -\frac{28}{9}$ (1) (1) (1/2)

[c] Find the **polar AND rectangular** coordinates of the $x-$ and $y-$ intercepts.

θ	r
0	$\frac{28}{5}$
$\frac{\pi}{2}$	-7
π	$\frac{28}{5}$
$\frac{3\pi}{2}$	2

X-INT POLAR $\left(\frac{28}{5}, 0\right)$ $\left(\frac{28}{5}, \pi\right)$ (1/2)

RECT $\left(\pm\frac{28}{5}, 0\right)$ (1/2)

Y-INT POLAR $\left(-7, \frac{\pi}{2}\right)$ $\left(2, \frac{3\pi}{2}\right)$ (1/2)

RECT $(0, -7)$ $(0, -2)$ (1/2)

[d] What are the rectangular coordinates of the vertices, center, foci, and endpoints of the latera recta ?

VERTICES:

$(0, -7)$ $(0, -2)$ (1)

CENTER:

$\left(0, \frac{-7-2}{2}\right) = \left(0, -\frac{9}{2}\right)$ (1)

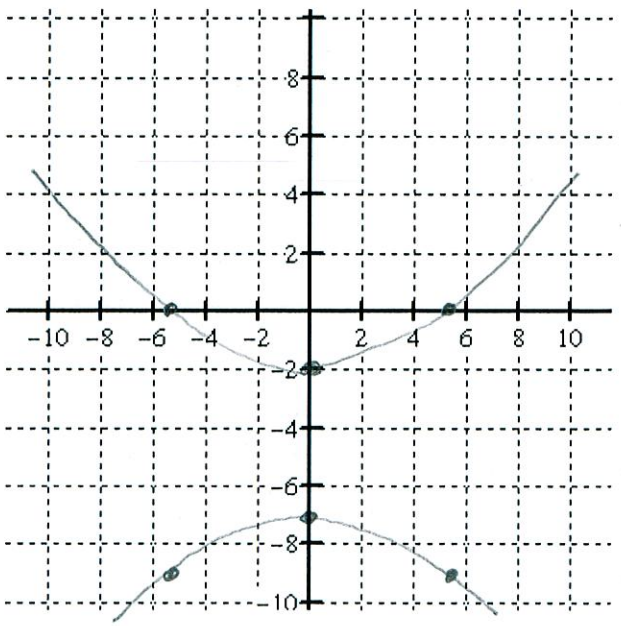
FOCI:

$\left(0, 2 \cdot \frac{-9}{2}\right) = (0, -9)$ (1) AND $(0, 0)$ (1/2)

ENDPOINTS OF LATERA RECTA:

$\left(\pm\frac{28}{5}, 0\right)$ (1) $\quad \left(\pm\frac{28}{5}, -9\right)$ (1)

[e] Graph the conic by connecting the relevant points from [d] appropriately.



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